JEE MAIN 2023

Paper with Solution

MATHS | 30th Jan 2023 _ Shift-2

Umeed Rank Ki Ho Ya Selection Ki,
JEET NISCHIT HAI!

Most Promising RANKS
Produced by MOTION Faculties

NEET / AIIMS

AIR-1 to 10
25 Times

AIR-11 to 50
83 Times

AIR-51 to 100
81 Times

JEE MAIN+ADVANCED

AIR-1 to 10
8 Times

AIR-11 to 50
32 Times

AIR-51 to 100
36 Times

Nation's Best SELECTION
Percentage (%) Ratio

Student Qualified in NEET

(2022)
4837/5356 = **90.31%**
(2021)
3276/3411 = **93.12%**

Student Qualified in JEE ADVANCED

(2022)
1756/4818 = **36.45%**
(2021)
1256/2994 = **41.95%**

Student Qualified in JEE MAIN

(2022)
4818/6653 = **72.41%**
(2021)
2994/4087 = **73.25%**

NITIN VIIJAY (NV Sir)
Founder & CEO

SECTION - A

61. A vector $\vec{v}$ in the first octant is inclined to the x-axis at 60°, to the y-axis at 45 and to the z-axis at an acute angle. If a plane passing through the points $(\sqrt{2}, -1, 1)$ and (a, b, c), is normal to $\vec{v}$, then

(1) $\sqrt{2}a + b + c = 1$ (2) $a + \sqrt{2}b + c = 1$

(3) $a + b + \sqrt{2}c = 1$ (4) $\sqrt{2}a - b + c = 1$

Sol. **2**

$\cos\alpha = \cos 60$ $\cos\beta = \cos 45$

$\ell = \frac{1}{2}$ $m = \frac{1}{\sqrt{2}}$

$l^2 + m^2 + n^2 = 1$

$\Rightarrow \frac{1}{4} + \frac{1}{2} + n^2 = 1$ $n^2 = 1 - \frac{3}{4} = \frac{1}{4}$

$\boxed{n = \frac{1}{2}}$

Direction of $\vec{v}$ is $\frac{1}{2}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{2}\hat{k}$

Equation of plane through ($\sqrt{2}$, –1, 1) & Normla to $\vec{v}$ is

$\frac{1}{2}(x - \sqrt{2}) + \frac{1}{\sqrt{2}}(y + 1) + \frac{1}{2}(z - 1) = 0$

It passes through (a,b,c)

$\left(a - \sqrt{2}\right) + \sqrt{2}(b + 1) + (c - 1) = 0$

$\Rightarrow a + \sqrt{2}b + c = \sqrt{2} - \sqrt{2} + 1$

$\Rightarrow \boxed{a + \sqrt{2}b + c = 1}$

62. Let $a, b, c > 1, a^3, b^3$ and c^3 be in A.P., and $\log_a b, \log_c a$ and $\log_b c$ be in G.P. If the sum of first 20 terms of an A.P., whose first term is $\frac{a+4b+c}{3}$ and the common difference is $\frac{a-8b+c}{10}$ is -444, then abc is equal to:

(1) $\frac{125}{8}$ (2) 216 (3) 343 (4) $\frac{343}{8}$

Sol. **2**

If $\log_a b, \log_c a, \log_b c \rightarrow$ G.P.

$(\log_c a)^2 = \log_a b \times \log_b c$

$(\log_c a)^2 = \log_a c$

$\Rightarrow (\log_c a)^2 = \frac{1}{\log_c a}$

$\Rightarrow (\log_c a)^3 = 1$

$\Rightarrow \log_c a = 1$

$\boxed{a = c}$

If $a^3b^3c^3 \to$ A.P

$2b^3 = a^3 + c^3$

If a = c

$\Rightarrow \boxed{a = b = c}$

For AP

$A = \dfrac{a+4a+a}{3}$ $\qquad$ $D = \dfrac{a-8a+a}{10}$

$A = 2a$ $\qquad$ $D = \dfrac{-3a}{5}$

$$S_{20} = \frac{20}{2}\left[2\times 2a + (20-1)\left(\frac{-3a}{5}\right)\right]$$

$$= 10\left[4a - \frac{57a}{5}\right]$$

$$= 10\left[-\frac{37a}{5}\right] = -444$$

$$\Rightarrow a = \frac{444\times 5}{37\times 10}$$

$$\boxed{a = 6}$$

$$\Rightarrow \boxed{abc = 6\times 6\times 6 = 216}$$

63. Let $a_1 = 1, a_2, a_3, a_4, ...$ be consecutive natural numbers.

Then $\tan^{-1}\left(\frac{1}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{1}{1+a_2a_3}\right) + \cdots. + \tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)$ is equal to

(1) $\cot^{-1}(2022) - \frac{\pi}{4}$

(2) $\frac{\pi}{4} - \cot^{-1}(2022)$

(3) $\tan^{-1}(2022) - \frac{\pi}{4}$

(4) $\frac{\pi}{4} - \tan^{-1}(2022)$

Sol. 3

$a_1 = 1, a_2, a_3, ...a_n$ be consecative natural numbers.

$$\tan^{-1}\left(\frac{1}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{1}{1+a_2a_3}\right) + ... + \tan^{-1}\left(\frac{1}{1+a_{2021}a_{2022}}\right)$$

$$\Rightarrow T_K = \tan^{-1}\left(\frac{1}{1+K(K+1)}\right)$$

$$= \tan^{-1}\left(\frac{K+1-K}{1+K(K+1)}\right)$$

$$= \tan^{-1}(K+1) - \tan^{-1}K$$

$T_1 = \tan^{-1}2 - \tan^{-1}1$

$T_2 = \tan^{-1}3 - \tan^{-1}2$

$T_3 = \tan^{-1}4 - \tan^{-1}3$

$\vdots$

$\underline{T_{2021} = \tan^{-1}(2022) - \tan^{-1}(2021)}$

On adding

$\Sigma T_n = \tan^{-1}(2022) - \tan^{-1}(1)$

$$\boxed{\sum_{n=1}^{2021} T_n = \tan^{-1}(2022) - \frac{\pi}{4}}$$

64. Let $\lambda \in \mathbb{R}, \vec{a} = \lambda\hat{i} + 2\hat{j} - 3\hat{k}, \vec{b} = \hat{i} - \lambda\hat{j} + 2\hat{k}$

If $((\vec{a}+\vec{b}) \times (\vec{a} \times \vec{b})) \times (\vec{a} - \vec{b}) = 8\hat{i} - 40\hat{j} - 24\hat{k}$, then $|\lambda(\vec{a}+\vec{b}) \times (\vec{a}-\vec{b})|^2$ is equal to

(1) 132 (2)136 (3) 140 (4) 144

Sol. **3**

$\left((\vec{a}+\vec{b})\times(\vec{a}\times\vec{b})\right)\times\left(\vec{a}-\vec{b}\right) - = 8\hat{i} - 40\hat{j} - 24\hat{k}$

$\Rightarrow \left(\vec{a}\times\left(\vec{a}\times\vec{b}\right) + \vec{b}\times\left(\vec{a}\times\vec{b}\right)\right)\times(\vec{a}-\vec{b})$

$\Rightarrow \left(\left(\vec{a}\cdot\vec{b}\right)\vec{a} - (\vec{a}\cdot\vec{a})\vec{b} + (\vec{b}\cdot\vec{b})\vec{a} - (\vec{b}\cdot\vec{a})\vec{b}\right)\times(\vec{a}-\vec{b})$

$\Rightarrow 0 - (\vec{a}\cdot\vec{b})(\vec{a}\times\vec{b}) - a^2(\vec{b}\times\vec{a}) + 0 - b^2(\vec{a}\times\vec{b}) - (\vec{a}\cdot\vec{b})\vec{b}\times\vec{a} = 8\hat{i} - 40\hat{j} - 24\hat{k}$

$\Rightarrow \left(a^2 - b^2\right)(\vec{a}\times\vec{b}) = 8\hat{i} - 40\hat{j} - 24\hat{k}$

$\left(\left(\lambda^2 + 4 + 9\right) - \left(1 + \lambda^2 + 4\right)\right)\left(\vec{a}\times\vec{b}\right)$

$8\left(\vec{a}\times\vec{b}\right) = 8(\hat{i} - 5\hat{j} - 3\hat{k})$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \lambda & 2 & -3 \\ 1 & -\lambda & 2 \end{vmatrix} = \hat{i} - 5\hat{j} - 3\hat{k}$$

$\hat{i}(4 - 3\lambda) - \hat{j}(2\lambda + 3) + \hat{k}(-\lambda^2 - 2) = \hat{i} - 5\hat{j} - 3\hat{k}$

$\Rightarrow$ $4 - 3\lambda = 1$ $\quad 2\lambda + 3 = 5$ $\quad -\lambda^2 - 2 = -3$

$3\lambda = 3$ $\quad \lambda^2 = 1$

$\boxed{\lambda = 1}$ $\quad \boxed{\lambda = 1}$

$\left|\lambda(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})\right| = \left|(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})\right|^2$

$\Rightarrow \left|-\vec{a}\times\vec{b} + \vec{b}\times\vec{a}\right|^2 = \left|2(\vec{a}\times\vec{b})\right|^2 = 4(1 + 25 + 9) = 140$

65. Let q be the maximum integral value of p in [0,10] for which the roots of the equation $x^2 - px + \frac{5}{4}p = 0$ are rational. Then the area of the region $\{(x, y): 0 \le y \le (x - q)^2, 0 \le x \le q\}$ is

(1) 243 (2) 164 (3) $\frac{125}{3}$ (4) 25

Sol. **1**

$x^2 - px + \frac{5}{4}p = 0$

Roots are rational

D = A perfect square

$p^2 - 4(1)\frac{5}{4}p$

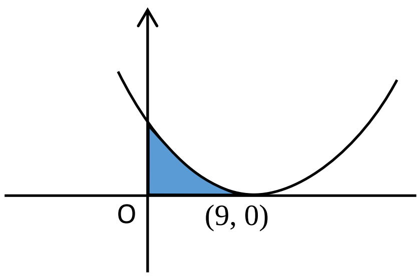
$p^2 - 5p$ = A perfect square

for p = 0, p = 5, p = 9 the D is a perfect square

$\therefore$ maximum integral of p is 9.

$\boxed{q = 9}$

$\{(x, y); 0 \le y \le (x-9)^2, 0 \le x \le 9\}$



Area = $\int_0^9 (x-9)^2 dx$

$\Rightarrow \left.\frac{(x-9)^3}{3}\right]_0^3$

$\Rightarrow 0 - \frac{(0-9)^3}{3}$

$\Rightarrow \frac{9 \times 9 \times 9}{3}$

= 243

66. Let f, g and h be the real valued functions defined on $\mathbb{R}$ as

$$f(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 1, & x = 0 \end{cases}, g(x) = \begin{cases} \frac{\sin(x+1)}{(x+1)}, & x \neq -1 \\ 1, & x = -1 \end{cases}$$

and $h(x) = 2[x] - f(x)$, where [x] is the greatest integer $\le$ x.

Then the value of $\lim_{x \to 1} g(h(x-1))$ is

(1) -1 (2) 0 (3) $\sin(1)$ (4) 1

Sol. **4**

LHL

$\lim_{\delta \to 0} g(h(-\delta)) \quad \delta > 0$

$\lim_{\delta \to 0} g(-2+1)$

$\Rightarrow g(-1) = 1$

RHL

$\lim\limits_{\delta \to 0} g(h(\delta))$

$\lim\limits_{\delta \to 0} g(2\times 0\text{-}1)$

$\lim\limits_{\delta \to 0} g(-1)$

$\lim\limits_{x \to 1} g(h(x-1)) = 1$

67. Let S be the set of all values of a_1 for which the mean deviation about the mean of 100 consecutive positive integers $a_1, a_2, a_3, \ldots . a_{100}$ is 25 . Then S is

(1) N (2) ϕ (3) $\{99\}$ (4) $\{9\}$

Sol. **1**

Let $a_1 = n$ $\quad a_2 = n+1$ $\quad a_3 = n+2$ $\quad \ldots\ldots$

$$\bar{x} = \frac{n+(n+1)+(n+2)+\ldots\ldots n+99}{100}$$

$$= \frac{100n + \frac{100\times 99}{2}}{100} = n + \frac{99}{2}$$

Mean deviation about the mean

$$\frac{1}{100}\sum |x_i - \bar{x}|$$

$$\Rightarrow \frac{1}{100}\left(\frac{99}{2}+\frac{97}{2}+\frac{95}{2}+\ldots\ldots.+\frac{97}{2}+\frac{99}{2}\right)$$

$$\Rightarrow \frac{2}{100}\left(\frac{99}{2}+\frac{97}{2}+\frac{95}{2}+\ldots\ldots.50\text{ terms}\right)$$

$$\Rightarrow \frac{2}{100}\times\frac{1}{2}\times(50)^2 = \frac{50\times 50}{100} = 25$$

It is 25 irrespective of the value of

$\therefore n \in N$

$\Rightarrow \boxed{S = N}$

68. For $\alpha, \beta \in \mathbb{R}$, suppose the system of linear equations

$x - y + z = 5$

$2x + 2y + \alpha z = 8$

$3x - y + 4z = \beta$

has infinitely many solutions. Then α and β are the roots of

(1) $x^2 + 14x + 24 = 0$ (2) $x^2 + 18x + 56 = 0$

(3) $x^2 - 18x + 56 = 0$ (4) $x^2 - 10x + 16 = 0$

Sol. **3**

$$\begin{vmatrix} 1 & -1 & 1 \\ 2 & 2 & \alpha \\ 3 & -1 & 4 \end{vmatrix} = 0$$

$1(8+\alpha)+1(8-3\alpha)+1(-2-6)=0$

$\Rightarrow\ 8+\alpha+8-3\alpha-8=0$

$-2\alpha=-8$

$\boxed{\alpha=4}$

$D_1 = 0$

$$\Rightarrow \begin{vmatrix} 5 & -1 & 1 \\ 8 & 2 & 4 \\ \beta & -1 & 4 \end{vmatrix} = 0$$

$5(8+4)+1(32-4\beta)+1(-8-2\beta)=0$

$60+32-4\beta-8-2\beta=0$

$\Rightarrow\ -6\beta=-84$

$\boxed{\beta=14}$

Equation having roots as $\alpha\ \&\ \beta$

$\boxed{x^2-18x+56=0}$

69. Let $\vec{a}$ and $\vec{b}$ be two vectors, Let $|\vec{a}| = 1, |\vec{b}| = 4$ and $\vec{a}\cdot\vec{b} = 2$. If $\vec{c} = (2\vec{a}\times\vec{b}) - 3\vec{b}$, then the value of $\vec{b}\cdot\vec{c}$ is

(1) -24 (2) -84 (3) -48 (4) -60

Sol. **3**

$\vec{b}.\vec{c} = (2\vec{a}\times\vec{b}).\vec{b} - 3\vec{b}.\vec{b}$

$= 0-3b^2$

$= -3 \times 16 = -48$

$\boxed{\vec{b}.\vec{c}=-48}$

70. If the functions $f(x) = \frac{x^3}{3} + 2bx + \frac{ax^2}{2}$ and $g(x) = \frac{x^3}{3} + ax + bx^2, a \neq 2b$ have a common extreme point, then $a + 2b + 7$ is equal to:

(1) $\frac{3}{2}$ (2) 3 (3) 4 (4) 6

Sol. **4**

$f'(x) = x^2+2b+ax=0$

$g'(x) = x^2+a+2bx=0$

$x = 1$ is the common root

$1 + 2b + a = 0$

$2b + a + 1 + 6 = 6$

$\boxed{2b+a+7=6}$

71. If P is a 3×3 real matrix such that $P^T = aP + (a-1)I$, where $a > 1$, then

(1) $|\text{Adj}P| = \frac{1}{2}$ (2) $|\text{Adj}P| = 1$

(3) P is a singular matrix (4) $|\text{Adj}P| > 1$

Sol. **2**

$$\left(P^T\right)^T = aP^T(a-1)I$$

$$P = a(aP + (a-1)I) + (a-1)I$$

$$= a^2P + (a^2 - a)I + (a-1)I$$

$$= a^2P + (a^2 - a + a - 1)I$$

$$P = a^2P + (a^2-1)I \quad \Rightarrow \quad P = (1-a^2) = (a^2-1)I$$

$$\boxed{P = -I}$$

$$|\text{Adj}P| = |P|^{3-1} = (-1)^2$$

$$= 1$$

72. The number of ways of selecting two numbers a and b, $a \in \{2, 4, 6, \ldots, 100\}$ and $b \in \{1, 3, 5, \ldots, 99\}$ such that 2 is the remainder when $a + b$ is divided by 23 is

(1) 268 (2) 108 (3) 54 (4) 186

Sol. **2**

a + b = 25,		a + b = 71		a + b = 117		a + b = 163	
a	b	a	b	a	b	a	b
2	23	2	69	18	99	64	99
4	21	4	67	20	97	66	97
⋮		⋮		⋮		⋮	
24	1	70	1	100	17	100	63
12 cases		35 cases		42 cases		19 cases	

Total ways = 12 + 35 + 42 + 19

= 108

73. $\lim\limits_{n\to\infty} \frac{3}{n}\left\{4 + \left(2+\frac{1}{n}\right)^2 + \left(2+\frac{2}{n}\right)^2 + \cdots + \left(3-\frac{1}{n}\right)^2\right\}$ is equal to

(1) 12 (2) $\frac{19}{3}$ (3) 0 (4) 19

Sol. **4**

$$\lim_{x\to\infty} \frac{3}{n}\left\{\underset{\substack{\uparrow \\ (2+0/n)^2}}{4} \; (2+1/n)^2 + \left(2+\frac{2}{n}\right)^2 + \ldots. + \left(3-\frac{1}{n}\right)^2\right\}$$

$$\Rightarrow \sum_{r=0}^{n-1} \frac{3}{n}\left(2+\frac{r}{n}\right)^2$$

$$\Rightarrow \int_0^1 3(2+x)^2 dx$$

$$\Rightarrow \left[(2+x)^3\right]_0^1$$

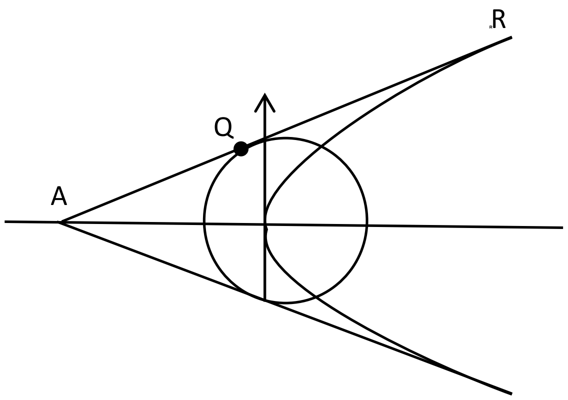
$$\Rightarrow (2+1)^3 - (2+0)^3$$

$$\Rightarrow 27-8=19$$

74. Let A be a point on the x-axis. Common tangents are drawn from A to the curves $x^2+y^2=8$ and $y^2=16x$. If one of these tangents touches the two curves at Q and R, then $(QR)^2$ is equal to

(1) 81 (2) 72 (3) 76 (4) 64

Sol. **2**



$y^2=16x$ circle

Tangent $x^2+y^2=8$

$y=mx+\frac{4}{m}$ Tagent

$y=mx\pm 2\sqrt{2}\sqrt{1+m^2}$

$\frac{4}{m}=\pm 2\sqrt{2}\sqrt{1+m^2}$

$\frac{16}{m^2}=8+8m^2$

$8m^4+8m^2=16$

$m^4+m^2=2$

$m^2=1,-2$

Let $m=1$

$\underline{m>1}$

$\therefore y=x+4$

Point of tangecy at parabola

$Q\left(\frac{4}{m^2},\frac{8}{m}\right)$

$Q(4,8)$

Point of tangency at circle

eq^n at tangent at $R(x_1, y_1)$

is $\boxed{T=0}$

$xx_1 + yy_1 = 8$

Comparison with

$x - y + 4 = 0$

$\frac{x_1}{1} = \frac{y_1}{-1} = -\frac{8}{4}$

$x_1 = -2 \qquad y_1 = 2$

$R(-2, 2)$

Now $QR^2 = \sqrt{(4+2)^2 + (8-2)^2}$

$\boxed{QR^2 = 72}$

75. If a plane passes through the points $(-1, k, 0), (2, k, -1), (1, 1, 2)$ and is parallel to the line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$, then the value of $\frac{k^2+1}{(k-1)(k-2)}$ is

(1) $\frac{17}{5}$ (2) $\frac{13}{6}$ (3) $\frac{6}{13}$ (4) $\frac{5}{17}$

Sol. **2**

Eq^n of plane

$$\begin{vmatrix} x-2 & y-k & 3+1 \\ 1-2 & 1-k & 2+1 \\ -1-2 & k-k & 0+1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-2 & y-k & 3+1 \\ -1 & 1-k & 3 \\ -3 & 0 & 1 \end{vmatrix} = 0$$

$(x-2)(1-k-0) - (y-k)(-1+9) + (3+1)(0+3-3k) = 0$

$\Rightarrow (1-k)x - 8y + (3-3k)z - 2 + 2k + 8k + 3 - 3k = 0$

$(1-K)x - 8y + (3-3K)z + 7K + 1 = 0$

Plane is parallel to the line L:

$\therefore (1-k)1 - 8.1 + (3-3K)(-1) = 0$

$\Rightarrow 1 - k - 8 - 3 + 3k = 0$

$2k = 10$

$\boxed{k=5}$

$\frac{k^2+1}{(k-1)(k-2)}$

$= \frac{25+1}{(5-1)(5-2)} = \frac{26}{4 \times 3} = \frac{13}{6}$

76. The range of the function $f(x) = \sqrt{3-x} + \sqrt{2+x}$ is:

(1) $[2\sqrt{2}, \sqrt{11}]$ (2) $[\sqrt{5}, \sqrt{13}]$ (3) $[\sqrt{2}, \sqrt{7}]$ (4) $[\sqrt{5}, \sqrt{10}]$

Sol. **4**

$3-x \geq 0$ $\quad$ $2+x \geq 0$

$x \leq 3$ $\quad$ $x \geq -2$

$x \in [-2, 3]$

Now, $f(-2) = \sqrt{3+2} = \sqrt{5}$

$f(3) = \sqrt{2+3} = \sqrt{5}$

$f(x) = \sqrt{3-x} + \sqrt{2+x}$

$f'(x) = \frac{1}{2\sqrt{3-x}} x - 1 + \frac{1}{2\sqrt{2+x}} = 0$

$\Rightarrow \frac{1}{\sqrt{3-x}} = \frac{1}{\sqrt{2+x}}$

$\Rightarrow 3-x = 2+x$

$\Rightarrow 2x = 1$

$x = 1/2$

$f\left(\frac{1}{2}\right) = \sqrt{3-1/2} + \sqrt{2+1/2}$

$= \sqrt{\frac{5}{2}} + \sqrt{\frac{5}{2}} \Rightarrow 2\sqrt{\frac{5}{2}} \Rightarrow \sqrt{10}$

Range $= \left[\sqrt{5}, \sqrt{10}\right]$

77. The solution of the differential equation $\frac{dy}{dx} = -\left(\frac{x^2+3y^2}{3x^2+y^2}\right), y(1) = 0$ is

(1) $\log_e|x+y| - \frac{xy}{(x+y)^2} = 0$ (2) $\log_e|x+y| + \frac{2xy}{(x+y)^2} = 0$

(3) $\log_e|x+y| - \frac{2xy}{(x+y)^2} = 0$ (4) $\log_e|x+y| + \frac{xy}{(x+y)^2} = 0$

Sol. **2**

$y = Vx$

$\frac{dy}{dx} = V + \frac{xdv}{dx}$

$V + \frac{xdv}{dx} = -\frac{x^2 + 3v^2x^2}{3x^2 + v^2x^2}$

$x\frac{dv}{dx} = -\frac{1+3v^2}{3+v^2} - v$

$x\frac{dv}{dx} = -\frac{1+3v^2+3v+v^3}{3+v^2}$

$\int \frac{3+v^2}{1+3v^2+3v+v^3} dv = -\int \frac{dx}{x}$

$$\Rightarrow \int \frac{3+v^2}{(1+v)^3} dv = -\ln x + C$$

Let $v+1=t$

$dv = dt$

$$\int \frac{3+(t-1)^2}{t^3} dt = -\ln x + c$$

$$\Rightarrow \int \frac{t^2-2t+4}{t^3} dt$$

$$\Rightarrow \int \left(\frac{1}{t} - \frac{2}{t^2} + \frac{4}{t^3}\right) dt = -\ln x + c$$

$$\Rightarrow \ln t + \frac{2}{t} - \frac{4}{2t^2} = -\ln x + C$$

$$\Rightarrow \ln\left(\frac{y}{x}+1\right)^{-1} \frac{2}{\frac{y}{x}+1} - \frac{4}{2\left(y/x+1\right)^2} = -\ln x + C$$

$$\Rightarrow \ln\left(\frac{y+x}{x}\right) + \frac{2x}{y+x} - \frac{2x^2}{(x+y)^2} = -\ln x + c$$

$$\Rightarrow \ln|x+y| + \frac{2x}{(x+y)^2}(x+y-x) = C$$

$$\Rightarrow \boxed{\ln|x+y| + \frac{2xy}{(x+y)^2} = C}$$

78. The parabolas : $ax^2 + 2bx + cy = 0$ and $dx^2 + 2ex + fy = 0$ intersect on the line $y = 1$. If a, b, c, d, e, f are positive real numbers and a, b, c are in G.P., then

(1) d, e, f are in G.P.

(2) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

(3) d, e, f are in A.P.

(4) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.

Sol. **2**

at $y=1$, Both curve intersect

$$\Rightarrow \left.\begin{matrix} ax^2+2bx+c=0 \\ dx^2+2ex+f=0 \end{matrix}\right\} \text{Common Root}$$

Given a, b, c are in G.P

$\boxed{b^2 = ac}$

$\Rightarrow D = 4b^2 - 4ac = 0$ for the first equation

$\Rightarrow$ Both the Root are equal

$\because$ sum of the roots $= -2\frac{b}{a}$

$$\alpha + \alpha = -2\frac{b}{a}$$

$$\boxed{\alpha = -\frac{b}{a}}$$

It satisfies the second equation also

$$d\left(-\frac{b}{a}\right)^2 + 2e\left(-\frac{b}{a}\right) + f = 0$$

$$d\left(\frac{b^2}{a^2}\right) - \frac{2eb}{a} + f = 0$$

$$d\left(\frac{ac}{a^2}\right) - 2e\frac{b}{a} + f = 0$$

$$\frac{d}{a} - \frac{2eb}{ac} + \frac{f}{c} = 0$$

$$\frac{d}{a} - \frac{2eb}{b^2} + \frac{f}{c} = 0$$

$$\Rightarrow \boxed{2\frac{e}{b} = \frac{d}{a} + \frac{f}{c}} \Rightarrow \boxed{\frac{d}{a}, \frac{e}{b}, \frac{f}{c} \text{ are in AP}}$$

79. Consider the following statements:

P : I have fever

Q: I will not take medicine

R : I will take rest.

The statement "If I have fever, then I will take medicine and I will take rest" is equivalent to:

(1) $((\sim P) \vee \sim Q) \wedge ((\sim P) \vee R)$

(2) $(P \vee Q) \wedge ((\sim P) \vee R)$

(3) $((\sim P) \vee \sim Q) \wedge ((\sim P) \vee \sim R)$

(4) $(P \vee \sim Q) \wedge (P \vee \sim R)$

Sol. **1**

$$P \to (\sim Q \wedge R)$$

$$\sim P V(\sim Q \wedge R)$$

$$\Rightarrow ((\sim P) V (\sim Q)) \wedge ((\sim P) v R)$$

80. $x = (8\sqrt{3} + 13)^{13}$ and $y = (7\sqrt{2} + 9)^9$. If $[t]$ denotes the greatest integer $\leq t$, then

(1) $[x]$ is odd but $[y]$ is even

(2) $[x] + [y]$ is even

(3) $[x]$ and $[y]$ are both odd

(4) $[x]$ is even but $[y]$ is odd

Sol. **2**

Let $x = I_1 + f_1$

$$\left(8\sqrt{3} + 13\right)^{13} = I_1 + f_1$$

$$\left(8\sqrt{3} - 13\right)^{13} = f_1'(\text{let})$$

On Subtaction

$\left(8\sqrt{3}+13\right)^{13}-\left(8\sqrt{b}-13\right)^{13}=I+f_1-f_1{}'$

$2\left[{}^{13}C_1\left(8\sqrt{3}\right)^{12}\right]13+{}^{13}C_3\left(8\sqrt{3}\right)^{10}13^3$

$+----]=I+0$

$\because I=\text{Even Number}$

$[x]=\text{Even}$

similarly,

Let $y=I_2+f_2$

$\left(7\sqrt{2}+9\right)^9=I_2+f_2$

$\left(7\sqrt{2}-9\right)^9=f_2'$

On Subtaction

$\left(7\sqrt{2}+9\right)^9-\left(7\sqrt{2}-9\right)^9=I_2+f_2-f_2^1$

$2\left[{}^9C_1\left(7\sqrt{2}\right)^8.9+{}^9C_3\left(7\sqrt{2}\right)^7 9^2---\right]=I_2+0$

$I_2=\text{Even}$

$$\boxed{\begin{aligned}\because [x]+[y]&=\text{Even}+\text{Even}\\&=\text{Even}\end{aligned}}$$

SECTION - B

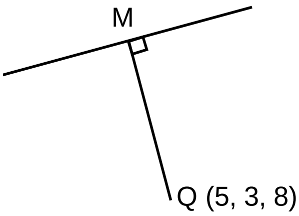
81. Let a line L pass through the point P(2,3,1) and be parallel to the line $x+3y-2z-2=0=x-y+2z$. If the distance of L from the point (5,3,8) is α, then $3\alpha^2$ is equal to ______.

Sol. **158**

The Direction ratio of line

$$\begin{vmatrix} i & j & k \\ 1 & 3 & -2 \\ 1 & -1 & 2 \end{vmatrix} \begin{aligned}&=i(6-2)-j(2+2)+k(-1-3)\\&=4i-4j-4k\end{aligned}$$

Equation of line L



$$\frac{x-2}{1}=\frac{y-3}{-1}=\frac{z-1}{-1}=\lambda(\alpha d)$$

Let M($\lambda+2, -\lambda+3, -\lambda+1$)

DR's of MQ is $<\lambda+2-5, -\lambda+3-3, -\lambda+1-8>$

$$< \lambda - 3, -\lambda . -\lambda > 7>$$

$\because$ $L \perp MQ$

$\Rightarrow (\lambda - 3)(1) + (-\lambda)(-1) + (-\lambda - 7)(-1) = 0$

$\Rightarrow \lambda - 3 + \lambda + \lambda + 7 = 0$

$\Rightarrow 3\lambda = -4 \Rightarrow \lambda = -\dfrac{4}{3}$

$\therefore\ M\left(-\dfrac{4}{3}+2, \dfrac{+4}{3}+3, \dfrac{4}{3}+1\right) = \left(\dfrac{2}{3}, \dfrac{13}{3}, \dfrac{7}{3}\right)$

$MQ = \alpha$

$\therefore\ 3\alpha^2 = 3 \times \left(\left(5-\dfrac{2}{3}\right)^2 + \left(3-\dfrac{13}{3}\right)^2 + \left(8-\dfrac{7}{5}\right)^2\right)$

$= 3\left(\dfrac{169}{9}+\dfrac{16}{9}+\dfrac{289}{9}\right) \Rightarrow \dfrac{474}{9} = 158$

82. A bag contains six balls of different colours. Two balls are drawn in succession with replacement. The probability that both the balls are of the same colour is p. Next four balls are drawn in succession with replacement and the probability that exactly three balls are of the same colour is q. If $p : q = m : n$, where m and n are coprime, then $m + n$ is equal to ______.

Sol. **14**

$p = 1.\dfrac{1}{6}$

$q = \left({}^6C_1 . \dfrac{1}{6} . \dfrac{1}{6} . \dfrac{1}{6} . \dfrac{5}{6}\right)\dfrac{4!}{3!} = \dfrac{5}{216} \times 4 = \dfrac{5}{54}$

$\dfrac{p}{q} = \dfrac{1/6}{5/54} = \dfrac{9}{5}$

$m = 9$

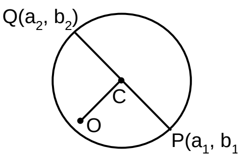
$n = 5$

$m + n = 9 + 5 = 14$

83. Let $P(a_1, b_1)$ and $Q(a_2, b_2)$ be two distinct points on a circle with center $C(\sqrt{2}, \sqrt{3})$. Let O be the origin and OC be perpendicular to both CP and CQ.

If the area of the triangle OCP is $\frac{\sqrt{35}}{2}$, then $a_1^2 + a_2^2 + b_1^2 + b_2^2$ is equal to ______.

Sol. **24**



OC is $\perp^r$ to both CP & CQ

$\Rightarrow$ PQ is a Diameter

Area of ΔOCP = $\dfrac{\sqrt{35}}{2}$

$\dfrac{1}{2}\times CP\times OC=\dfrac{\sqrt{35}}{2}$

CP $\times$ $\sqrt{2+3}=\sqrt{35}$

CP = $\sqrt{7}$ $\Rightarrow$ radius = $\sqrt{7}$

Now $OP^2 = OC^2 + PC^2$

$a_1^2+b_1^2=2+3+7=12$

Similarly

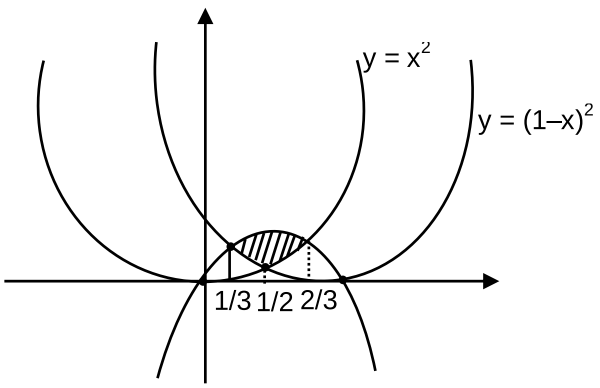
$OQ^2 = OC^2 + CQ^2$

$a_1^2+b_1^2=2+3+7=12$

$\therefore\ a_1^2+a_2^2+b_1^2+b_2^2=24$

84. Let A be the area of the region $\{(x, y): y \geq x^2, y \geq (1-x)^2, y \leq 2x(1-x)\}$. Then 540 A is equal to ______.

Sol. **25**



$x^2 = (1-x)^2$

$x^3 = 1 + x^2 - 2x$

$x = \dfrac{1}{2}$

$x^2 = 2x - 2x^2$

$3x^1 = 2x$

$x(3x-2) = 0$

$x = 0, \dfrac{2}{3}$

$(1-x)^2 + 2x - 2x^2$

$1 + x^2 - 2x = 2x - 2x^2$

$\Rightarrow 3x^2 - 4x + 1 = 0$

$\Rightarrow 3x^2 - 3x - x + 1 = 0$

$\Rightarrow 3x(x-1) - 1(x-1) = 0$

$x = 1, \dfrac{1}{3}$

Required Area

$$A=\int_{\frac{1}{3}}^{\frac{1}{2}}\left\{(2x-2x^2)-(1-x)^2\right\}dx+\int_{\frac{1}{2}}^{\frac{2}{3}}\left((2x-2x^2)-x^2\right)dx$$

$$\Rightarrow \left[x^2-\frac{2x^3}{3}+\frac{(1-x)^3}{3}\right]_{\frac{1}{3}}^{\frac{1}{2}}+(x^2-x^3)_{1/2}^{2/3}$$

$$\Rightarrow \left(\frac{1}{4}-\frac{2}{3}.\frac{1}{8}+\frac{1}{8.3}\right)-\left(\frac{1}{9}-\frac{2}{3}.\frac{1}{27}+\frac{8}{27.3}\right)+\left(\frac{4}{9}-\frac{8}{27}\right)-\left(\frac{1}{4}-\frac{1}{8}\right)$$

$$\Rightarrow -\frac{1}{24}-\frac{1}{9}-\frac{6}{3\times 27}+\frac{4}{9}-\frac{8}{27}+\frac{1}{8}$$

$$\Rightarrow -\frac{1}{24}+\frac{3}{9}-\frac{10}{27}+\frac{3}{24}=\frac{-27+216-240+81}{24\times 27}=\frac{297-267}{24\times 27}=A$$

$$540\text{ A} = 540\times\frac{30}{24\times 27}=25$$

85. The 8^{th} common term of the series

$$S_1 = 3+7+11+15+19+\cdots..$$
$$S_2 = 1+6+11+16+21+\cdots.$$

is _______.

Sol. **151**

8^{th} common term of the series

$S_1 = 3 + 7 + 11 + 15 + 19 + \ldots\ldots\ldots$

$S_2 = 1 + 6 + 11 + 16 + 21 + \ldots\ldots\ldots$

First common term = 11

common diff of the AP of common terms

= L.C.M of {4, 5}

= 20

$\therefore$ AP

11, 31, 51, …..

$T_8 = 11 + (8 - 1)20$

$=11 + 140$

$T_8 = 151$

86. Let $A = \{1,2,3,5,8,9\}$. Then the number of possible functions $f: A \to A$ such that $f(m\cdot n) = f(m)\cdot f(n)$ for every $m, n \in A$ with $m \cdot n \in A$ is equal to _______.

Sol. **1**

LHL	RHL
$\lim\limits_{h\to 0} g(H(1-h-1))$	$\lim\limits_{h\to 0} g(H(1+h-1))$
$\lim\limits_{h\to 0} g(2(-1)-f(-h))$	$\lim\limits_{h\to 0} g(H(h))$
$\lim\limits_{h\to 0} g\left(-2-\frac{1-h}{\lvert -h\rvert}\right)$	$\Rightarrow \lim\limits_{h\to 0} g(2(0)+(h))$
$\Rightarrow g\left(-2-\frac{-1}{1}\right)$	g (0 – 1)
$\Rightarrow 2(-1) = +1$	$\Rightarrow 1$

$\therefore \lim\limits_{h\to 0} g(H(x-1)=1$

87. If $\int \sqrt{\sec 2x - 1}\,dx = \alpha \log_e \left|\cos 2x + \beta + \sqrt{\cos 2x\left(1 + \cos \frac{1}{\beta}x\right)}\right| + \text{constant}$, then $\beta - \alpha$ is equal to ______.

Sol. **1**

$$I = \int \sqrt{\sec 2x - 1}\,dx$$

$$\Rightarrow \int \sqrt{\frac{1-\cos 2x}{\cos 2x}}\,dx$$

$$\Rightarrow \int \frac{\sqrt{2}\sin x}{\sqrt{2\cos^2 x - 1}}\,dx$$

Let $\sqrt{2}\cos x = t$

$-\sqrt{\alpha}\sin x\; dx = dt$

$$I = \int \frac{-dt}{\sqrt{t^2 - 1}} = \ln\left|t + \sqrt{t'-1}\right| + c$$

$$\Rightarrow -\ln\left|\sqrt{2}\cos x + \sqrt{2\cos^2 x - 1}\right| + c$$

$$\Rightarrow \frac{-1}{2}\ln\left|\left(\sqrt{2}\cos x + \sqrt{\cos 2x}\right)^2\right| + c$$

$$\Rightarrow \frac{-1}{2}\ln\left|2\cos^2 x + \cos 2x + 2\sqrt{2}\cos x\sqrt{\cos 2x}\right| + c$$

$$\Rightarrow \frac{-1}{2}\ln\left|1 + \cos 2x + \cos 2x + 2\sqrt{2}\sqrt{\cos 2x} \times \sqrt{\frac{1+\cos x}{2}}\right| + c$$

$$\Rightarrow \frac{-1}{2}\ln\left|2\cos 2x + 1 + 2\sqrt{\cos 2x\left(1 + \cos 2x\right)}\right| + c$$

$$\Rightarrow \frac{-1}{2}\ln\left|\cos 2x + \frac{1}{2}\sqrt{\cos 2x\left(1 + \cos 2x\right)}\right| + c$$

$$\alpha = \frac{-1}{2} \qquad \beta = \frac{1}{2}$$

$$\therefore \boxed{\beta - \alpha = \frac{1}{2} - \left(\frac{-1}{2}\right) = 1}$$

88. If the value of real number $a > 0$ for which $x^2 - 5ax + 1 = 0$ and $x^2 - ax - 5 = 0$ have a common real root is $\frac{3}{\sqrt{2\beta}}$ then β is equal to ______.

Sol. **13**

$$x^2 - 5ax + 1 = 0$$

$$x^2 - ax - 5 = 0$$

$$\underline{-\quad +\quad +}$$

$$-4ax + 6 = 0$$

$x = \frac{6}{4a} = \frac{3}{2a}$ (common root)

$\therefore \left(\frac{3}{2a}\right)^2 - 5a\left(\frac{3}{2a}\right) + 1 = 0$

$\Rightarrow 9 - 30a^2 + 4a^2 = 0$

$\Rightarrow 26a^2 = 9$

$a^2 = \frac{9}{26} \Rightarrow a = \frac{3}{\sqrt{26}} = \frac{3}{\sqrt{2\beta}}$

$\beta = 13$

89. 50^{th} root of a number x is 12 and 50^{th} root of another number y is 18 . Then the remainder obtained on dividing $(x + y)$ by 25 is ______.

Sol. **23**

$x^{\frac{1}{50}} = 12$ $\qquad y^{\frac{1}{50}} = 18$

Remainder when x + y is division by 25.

$x = 12^{50}$ $\qquad y = 18^{50}$

$x + y = 12^{50} + 18^{50}$

$= 6^{50}(2^{50} + 3^{50})$

$= (5 + 1)^{50}\ ((2^2)^{25} + (3^2)^{25})$

$= (25\lambda_1 + 1)\ ((5-1)^{25} + (10-1)^{25})$

$= (25\lambda_1 + 1)\ (25\ (\lambda_2 + \lambda_3) - 2)$

$= (25\lambda_1 + 1)\ (25\ K - 2)$

$\Rightarrow 25\lambda_1 \,.\, 25K - 50\lambda_1 + 25K - 2$

$\Rightarrow 25n_1 - 2$

$\Rightarrow 25n_2 + 23$

Remainder = 23

90. The number of seven digits odd numbers, that can be formed using all the seven digits 1,2,2,2,3,3,5 is ______.

Sol. **240**

The no. of 7 digit odd Numbers that can be formed using

1, 2, 2, 2, 3, 3, 5

$\square\square\square\square\square\square\boxed{1}\quad \frac{|\!\underline{6}}{|\!\underline{3}\,|\!\underline{2}} = \frac{720}{12} = 60$

$\square\square\square\square\square\square\boxed{3}\quad \frac{|\!\underline{6}}{|\!\underline{3}} = \frac{720}{6} = 120$

$\square\square\square\square\square\square\boxed{5}\quad \frac{|\!\underline{6}}{|\!\underline{3}\,|\!\underline{2}} = \frac{720}{12} = 60$

= 240


मोशन है, तो
भरोसा है।
#RankBhiSelectionBhi
ADMISSION ANNOUNCEMENT
Session 2023-24 (English & हिन्दी Medium)
Target: JEE/NEET 2025
Nurture & प्रयास Batch
Class 10th to 11th Moving
Target: JEE/NEET 2024
Enthuse & प्रयास Batch
Class 11th to 12th Moving
Target: JEE/NEET 2024
Dropper & प्रयास Batch
Class 12th to 13th Moving
Target: PRE FOUNDATION
SIP, Evening & Tapasya Batch
Class 6th to 10th Students
MOTION